

Definitions

- **Network** – Any structure containing interconnected elements.
- **Circuit** – Usually physical structure constructed from electrical components.

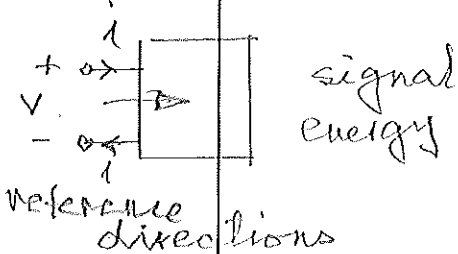
(A) **Linear Network:** response proportional to excitation. Superposition applies:

| | |
|------|---|
| | $If \ e_1(t) \rightarrow w_1(t) \text{ and } e_2(t) \rightarrow w_2(t)$ |
| Then | |
| | $k_1 \cdot e_1(t) + k_2 \cdot e_2(t) \rightarrow k_1 \cdot w_1(t) + k_2 \cdot w_2(t)$ |

ideal only

(B) **Time-Invariant Network:** $e(t) \rightarrow w(t)$ relation the same if $t \rightarrow t + t_1$. Time varying otherwise.

(C) **Passive Network:** EM energy delivered always non-negative. Specifically:

| | | |
|---|---|--|
| | $E(t) = \int_{-\infty}^t \underbrace{v(x)i(x)}_P dx \geq 0$ | |
| or | <i>associated directions</i> | |
| | $E(t) = \int_{t_0}^t v(x)i(x) dx + E(t_0) \geq 0$ | |
| | |  |
| This must be true for any voltage and its resulting current for all t . | | |

Otherwise, active.

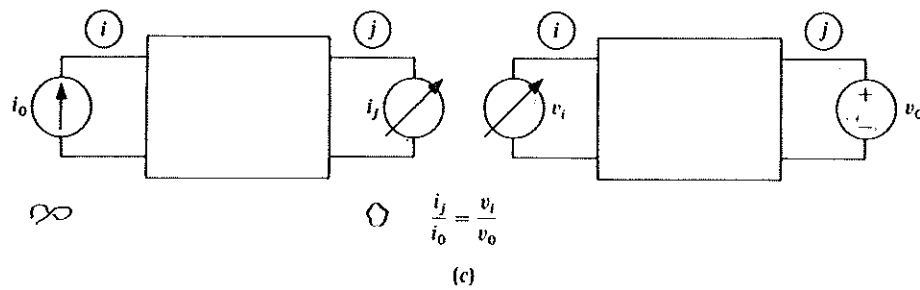
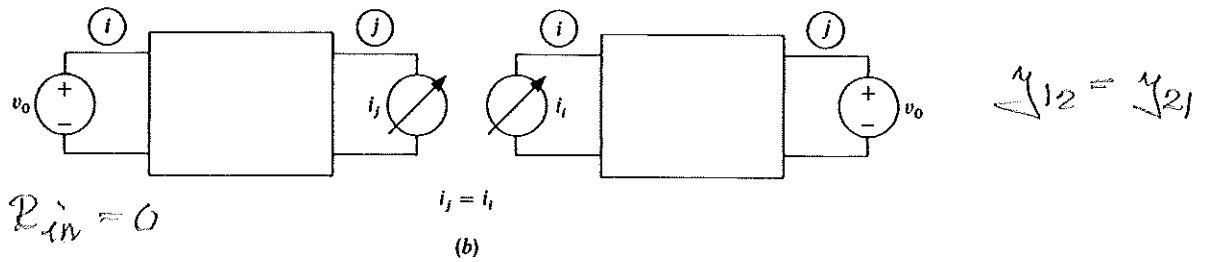
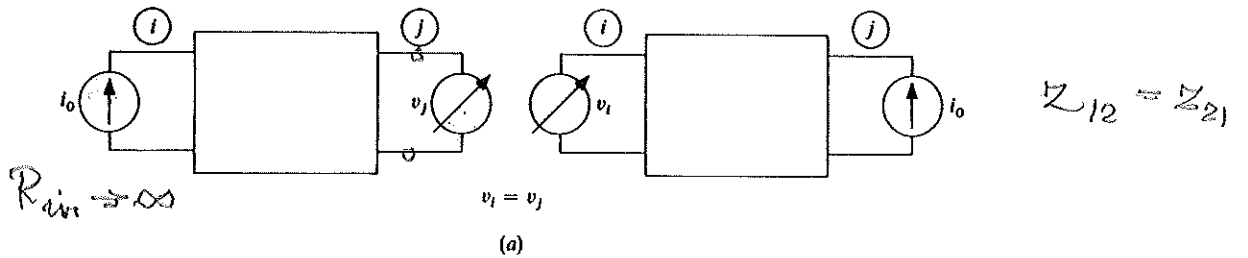
(D) **Lossless Circuit:** input energy is always equal to the energy stored in the network. Otherwise, lossy.

(E) **Distributed Network:** must use Maxwell's equation to analyze. Examples: transmission lines, high speed VLSI circuits, PCBs.

lumped

(F) **Memoryless or Resistivity^e Circuit:** no energy storing elements. Response depends only on instantaneous excitation. Otherwise, dynamic or memoried circuit.

(G) **Reciprocity:** response remains the same if excitation and response locations are interchanged. Specifically:

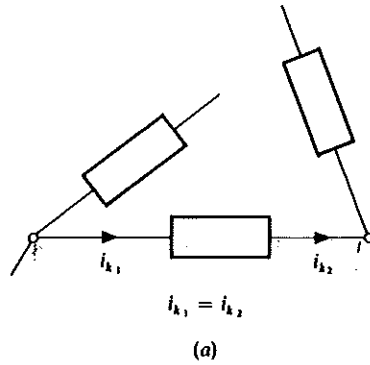


Otherwise, non-reciprocal.

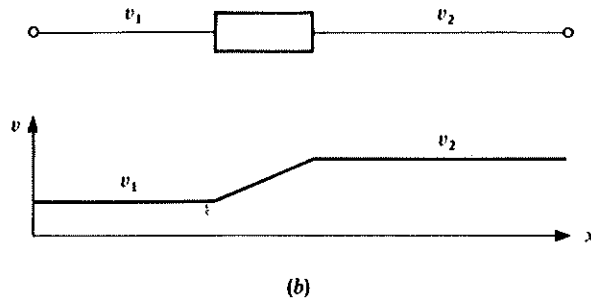
$$h_{12} = -h_{21}$$

$$g_{12} = -g_{21}$$

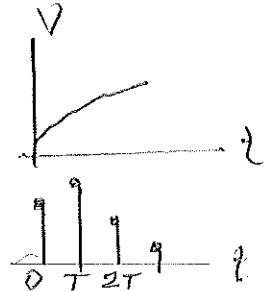
- (H) **Lumped Network:** physical dimensions can be considered zero. In reality, much smaller than the wavelength of the signal.



KCL
 KVL
 Branch eqs.






- (I) **Continuous-Time Circuit:** the signals can take on any value at any time.
Discrete-Time
- (J) **Sampled-Data Circuit:** the signals have a known value only at some discrete time instances. Digital, analog circuits.



An ideal RLC circuit is linear, time-invariant, passive, lossy, reciprocal, lumped, dynamic continuous-time network.

(A) Ideal R, L, C: ≥ 0

| Element | Parameter | Voltage-Current Relationship | | Symbol |
|-----------|--|------------------------------|--|---|
| | | Direct | Inverse | |
| Resistor | Resistance R Conductance G | $v = Ri$ | $i = \frac{1}{R}v = Gv$ |  |
| Inductor | Inductance L Inverse Inductance T | $v = L \frac{di}{dt}$ | $i(t) = \frac{1}{L} \int_0^t v(x) dx + i(0)$ |  |
| Capacitor | Capacitance C Elastance D | $i = C \frac{dv}{dt}$ | $v(t) = \frac{1}{C} \int_0^t i(x) dx + v(0)$ |  |

duals

Associated directions for v & i

Table 1

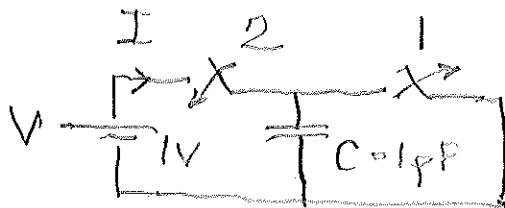
Each passive;

Assuming standard references, the energy delivered to each of the elements starting at a time when the current and voltage were zero will be:

$$E_R(t) = \int_{-\infty}^t Ri^2(x) dx \geq 0 \quad R > 0 \quad (67)$$

$$E_L(t) = \int_{-\infty}^t L \frac{di(x)}{dx} i(x) dx = \int_0^{i(t)} Li di = \frac{1}{2} Li^2(t) \geq 0 \quad (68)$$

$$E_C(t) = \int_{-\infty}^t C \frac{dv(x)}{dx} v(x) dx = \int_0^{v(t)} Cv dv = \frac{1}{2} Cv^2(t) \geq 0 \quad (69)$$



"Home work 1"

$E_C = ?$ stored energy

$E_V = ?$ Delivered energy

Port
 Two-port

(B) Ideal Transformer:

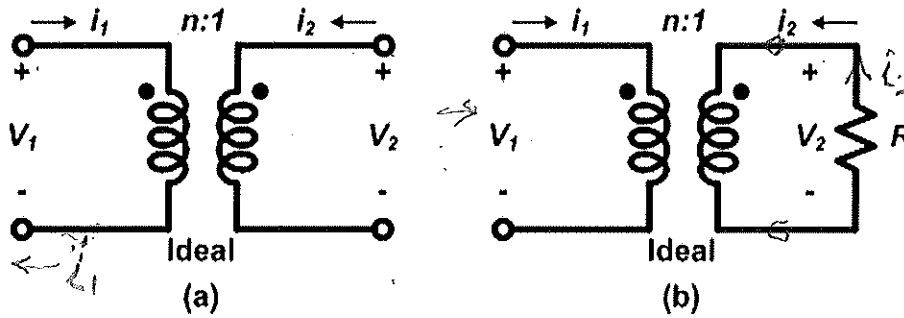


Fig. 6 An ideal transformer

Defined in terms of the following v-i relationships:

$$v_1 = n v_2 \quad \text{turn ratio} \quad (70a)$$

$$i_2 = -n i_1 \quad (70b)$$

or

$$\begin{bmatrix} v_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 0 & n \\ -n & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ v_2 \end{bmatrix} \quad (70c)$$

$$v_1 = n v_2 = -n R i_1 = (n^2 R) i_1 \quad (71)$$

$$R_{in} = v_1 / i_1 = n^2 R$$

At the input terminals, then, the equivalent resistance is $n^2 R$. Observe that the total energy delivered to the ideal transformer from connections made at its terminals will be

$$E(t) = \int_{-\infty}^t \underbrace{(v_1(x) i_1(x) + v_2(x) i_2(x))}_{P = 0} dx = 0 \quad (72)$$

Lossless, memoryless!

The right-hand side results when the v-i relations of the ideal transformer are inserted in the middle. Thus, the device is passive; it transmits, but neither stores nor dissipates energy.

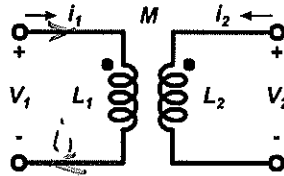
Memoryless!

(C) ¹¹Physical Transformer:

L_1 : primary self-inductance

M : mutual inductance

Henry



Two-port

Fig. 7 A transformer

The diagram is almost the same except that the diagram of the ideal transformer shows the turns ratio directly on it. The transformer is characterized by the following v-i relationships for the reference shown in Fig. 7:

$$v_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} \quad \text{mutual ind.} \quad (73a)$$

And

primary self-ind.

$$v_2 = M \frac{di_1}{dt} + L_2 \frac{di_2}{dt} \quad (73b)$$

Thus it is characterized by three parameters: the two self-inductances L_1 and L_2 , and the mutual inductance M . The total energy delivered to the transformer from external sources is

$$E(t) = \int_{-\infty}^t [v_1(x)i_1(x) + v_2(x)i_2(x)] dx$$

$$= \int_0^{i_1} L_1 i_1' di_1' + \int_0^{i_2} M d(i_1' i_2') + \int_0^{i_2} L_2 i_2' di_2' \quad (74)$$

$\times \frac{d}{dt} i_1/i_2$

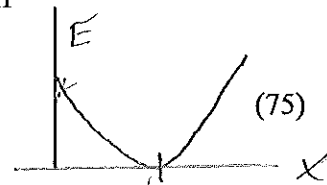
$$= \frac{1}{2} (L_1 i_1^2 + 2M i_1 i_2 + L_2 i_2^2) \geq 0 \quad \text{due to physical considerations!}$$

$$L_1 x^2 + 2Mx + L_2$$

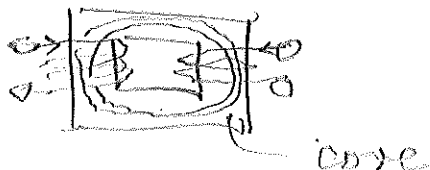
It is easy to show that the last line will be non-negative if

" Homework 2 "

$$\frac{M^2}{L_1 L_2} = k^2 \leq 1$$



Since physical considerations require the transformer to be passive, this condition must apply. The quantity k is called the *coefficient of coupling*. Its maximum value is unity for a closely-coupled transformer.



A transformer for which the coupling coefficient takes on its maximum value $k = 1$ is called a *perfect*, or *perfectly coupled*, transformer. A perfect transformer is not the same thing as an ideal transformer. To find the difference, turn to the transformer equations (73) and insert the perfect-transformer condition $M = \sqrt{L_1 L_2}$; then take the ratio v_1/v_2 . The result will be

$$\frac{v_1}{v_2} = \frac{L_1 \frac{di_1}{dt} + \sqrt{L_1 L_2} \frac{di_2}{dt}}{\sqrt{L_1 L_2} \frac{di_1}{dt} + L_2 \frac{di_2}{dt}} = \sqrt{L_1/L_2}. \quad (76)$$

This expression is identical with $v_1 = n v_2$ for the ideal transformer† if

$$n = \sqrt{L_1/L_2}. \quad (77)$$

Next let us consider the current ratio. Since (73) involve the derivatives of the currents, it will be necessary to integrate. The result of inserting the perfect-transformer condition $M = \sqrt{L_1 L_2}$ and the value $n = \sqrt{L_1/L_2}$, and integrating (73a) from 0 to t will yield, after rearranging,

$$i_1(t) = -\frac{1}{n} i_2(t) + \left\{ \frac{1}{L_1} \int_0^t v_1(x) dx + \left[i_1(0) + \frac{1}{n} i_2(0) \right] \right\}. \quad (78)$$

This is to be compared with $i_1 = -i_2/n$ for the ideal transformer. The form of the expression in brackets suggests the v - i equation for an inductor. The diagram shown in Fig. 8 satisfies both (78) and (76). It shows how a perfect transformer is related to an ideal transformer. If, in a perfect transformer, L_1 and L_2 are permitted to approach infinity, but in such a way that their ratio remains constant, the result will be an ideal transformer.

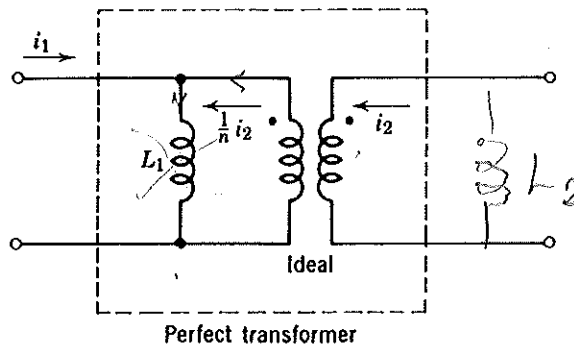


Fig. 8. Relationship between a perfect and an ideal transformer.

Lossless, memoried element.

(D) The Gyrator:

Definitions:

- **Port:** Two terminals, both input leads always carrying the same current.
- **Gyrator:** A two port network requiring active components for realization.

gyration res.

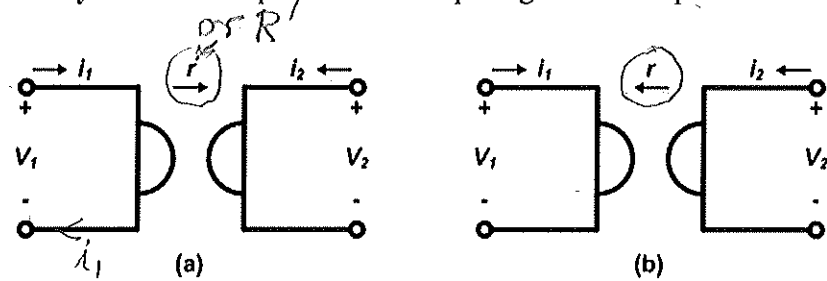


Fig. 9 A gyrator

Two-port

$V(j\omega) = 1 + j2$

Often used to transform (convert) impedance into a different kind. Generally,

$Z_{in} = \frac{r^2}{Z_{load}}$ *jw or s domain*

Time domain analysis
 $v_1(t)$

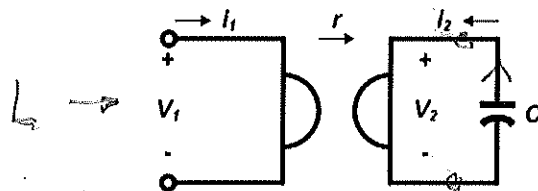
For Fig. 9(a) $V_1 = -ri_2$ or $\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 0 & -r \\ r & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$ (79a)

For Fig. 9(b) $V_1 = ri_2$ or $\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 0 & r \\ -r & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$ (79b)

s or jw
 $Z_{in} = \frac{r^2}{Z}$

$E(t) = \int_{-\infty}^t (v_1 i_1 + v_2 i_2) dx = \int_{-\infty}^t [(-ri_2)i_1 + (ri_1)i_2] dx = 0$ (80)

Lossless
Memoryless
Non-reciprocal



$i_2 = -C \frac{dv_2}{dt}$

Fig. 11 Gyrator terminated in a capacitor C

$i_2 = -C \frac{dv_2}{dt}$. Therefore, upon inserting the v-i relations associated with the gyrator, we observe that

$\underline{v_1} = -ri_2 = -r \left(-C \frac{dv_2}{dt} \right) = rC \frac{d(ri_1)}{dt} = r^2 C \frac{di_1}{dt} \triangleq L \frac{di_1}{dt}$ (82)

$L \triangleq r^2 C$

$r = 1 \text{ k}\Omega$

$i_1 = 1 \mu\text{A} \rightarrow V_2 = 1\text{V}$

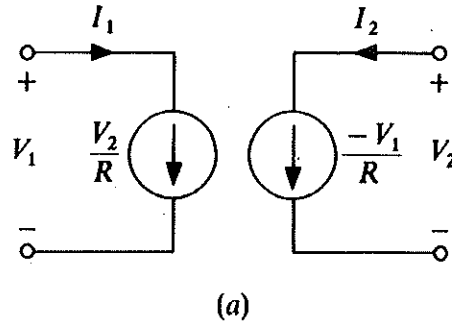
$i_2 = 1 \mu\text{A} \rightarrow V_1 = -1\text{V}$

non-reciprocal!

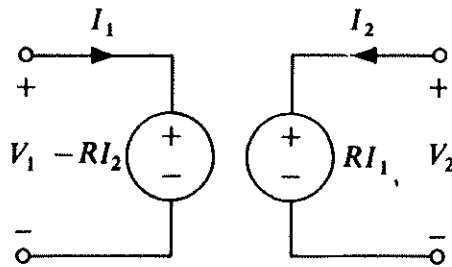
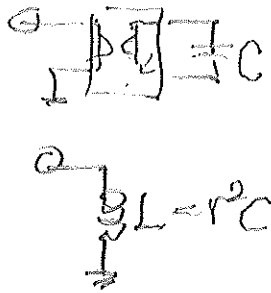
Philips filters

(The first one is more practical, using transconductors)

$r = R$

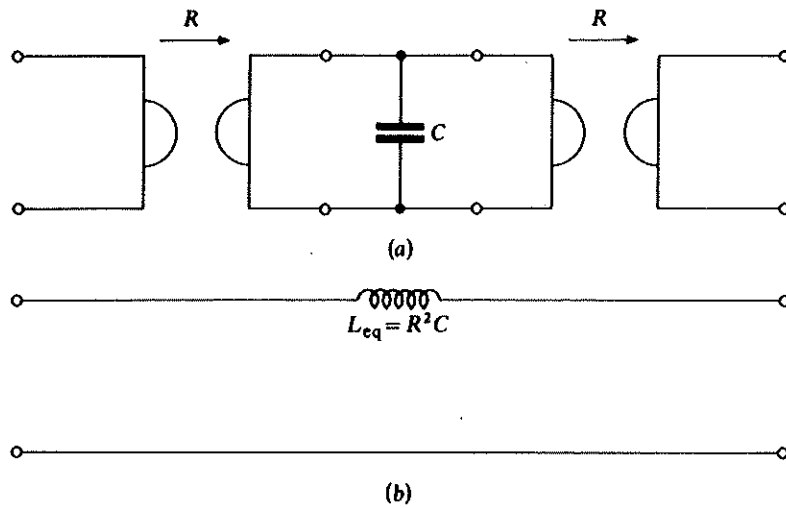


VCCS
 Transconductor
 G_M



CCVS

Figure 7-18 Ideal gyration circuit



"Homework 3"

Figure 7-24 Floating-inductor simulation using gyration

The Riordan circuit using two op-amps:

Laplace domain!

$$Z = \frac{V(s)}{I(s)}$$

$s = j\omega \rightarrow$ phasor

$v = 3 \sin 2\omega t$
 $i = \cos 2\omega t$
 $Z \neq 3 \tan 2\omega t$

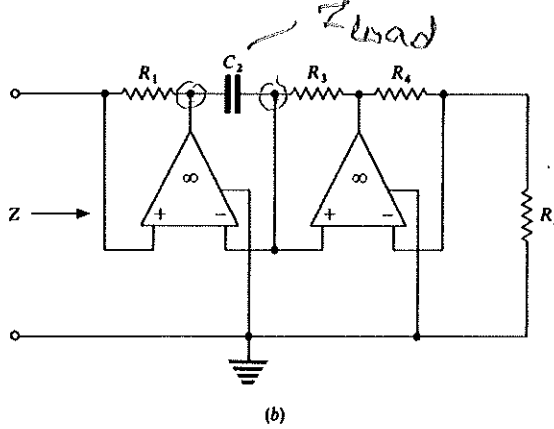
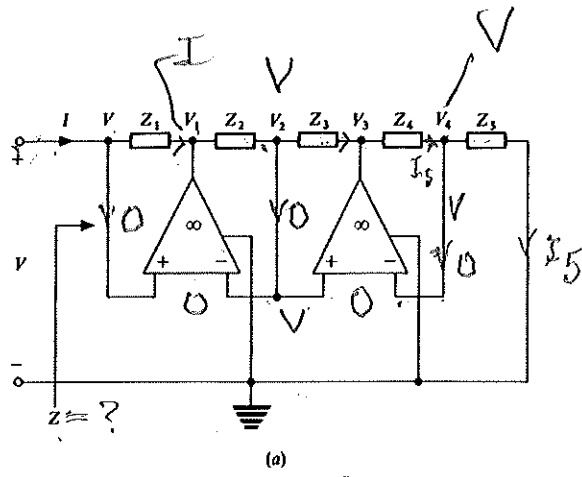
What is $Z(j\omega)$?

"Homework 4"

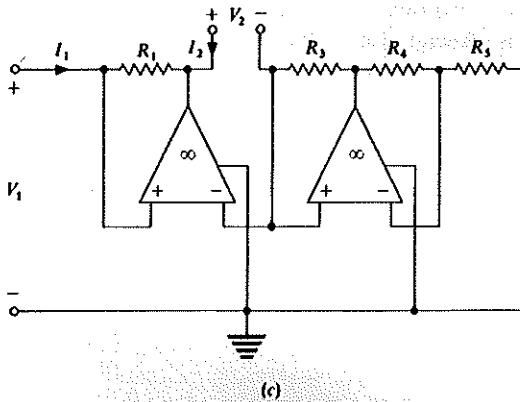
$$I_5 = \frac{V}{Z_5}$$

GIC

opamps must be stable, ideal
 CMRR !?



Floating opamps
 $V_{CM} = V$
 Linearity!



gyrator

Figure 7-19 The Riordan circuit: (a) basic circuit; (b) used as an inductor; (c) used as a gyrator

A circuit which uses two grounded-output op-amps and is useful for the realization of either GICs or GIIs is shown in Fig. 7-19a.† The input impedance Z can easily be found, as follows. When we recall that the input voltage of an op-amp is very nearly zero,

$$V \approx V_2 \approx V_4 \quad (7-62)$$

is obtained. Also, if we denote the current through Z_1 by I_1 (with the reference direction pointing left to right), the current through Z_2 by I_2 , etc., clearly

$$\begin{aligned} I_1 \approx I \quad V - V_1 = I_1 Z_1 \approx V_2 - V_1 = -I_2 Z_2 \\ I_3 \approx I_2 \quad V_2 - V_3 = I_3 Z_3 \approx V_4 - V_3 = -I_4 Z_4 \\ I_5 \approx I_4 \quad V \approx V_4 = I_5 Z_5 \end{aligned} \quad (7-63)$$

Here we assumed, as usual, that the current in the input leads of the op-amps is zero.

Working backward in (7-63) leads to

$$V \approx I_5 Z_5 \approx I_4 Z_5 \approx -I_3 \frac{Z_3}{Z_4} Z_5 \approx -I_2 \frac{Z_3}{Z_4} Z_5 \approx I_1 \frac{Z_1 Z_3}{Z_2 Z_4} Z_5 \approx I \frac{Z_1 Z_3 Z_5}{Z_2 Z_4} \quad (7-64)$$

Hence
$$Z = \frac{V}{I} \approx \frac{Z_1 Z_3 Z_5}{Z_2 Z_4} \quad (7-65)$$

If Z_5 is regarded as a load impedance, the circuit behaves like a GIC; (7-46) takes the form

$$Z(s) = f(s) Z_5(s) \quad f(s) \equiv \frac{Z_1(s) Z_3(s)}{Z_2(s) Z_4(s)} \quad (7-66)$$

If, for example, $Z_1 = R_1$, $Z_2 = 1/sC_2$, $Z_3 = R_3$, $Z_4 = R_4$, and $Z_5 = R_5$ (Fig. 7-19b), then $f(s) = R_1 R_3 / [(1/sC_2) R_4]$ and

$$Z = \frac{R_1 R_3}{(1/sC_2) R_4} R_5 = s \frac{R_1 C_2 R_3 R_5}{R_4} \approx s L \quad (7-67)$$

Hence, the input impedance is that of an *inductor*, with an equivalent inductance value $L_{eq} = R_1 C_2 R_3 R_5 / R_4$.

As (7-67) suggests, and as can be directly verified from (7-65), the two-port formed by regarding the terminals of Z_2 as an output port is a *gyrator* if all other impedances are purely resistive (Fig. 7-19c). More generally, if the terminals of Z_5 (or Z_1 or Z_3) constitute the output port, the circuit of Fig. 7-19a is a GIC; if the terminals of Z_2 (or Z_4) form the output port, the resulting two-port is a GII.

Assume now that we choose Z_2 and Z_4 as capacitive and Z_1 , Z_3 , and Z_5 as resistive impedances. Then (7-65) gives, for $s = j\omega$,

$$Z(j\omega) = \frac{R_1 R_3 R_5}{(1/j\omega C_2)(1/j\omega C_4)} = -\omega^2 R_1 C_2 R_3 C_4 R_5 \quad (7-68)$$

We note that $Z(j\omega)$ is pure real, negative, and a function of ω . Such an impedance† is called a *frequency-dependent negative resistance* (FDNR). A slightly different form of FDNR can be obtained, e.g., by choosing Z_1 and Z_3 as capacitors and Z_2 , Z_4 , and Z_5 as resistors. Then

$$Z(j\omega) = -\frac{R_5}{C_1 R_2 C_3 R_4 \omega^2} \quad (7-69)$$

As we shall see later, FDNRs are very useful for the design of active filters.